

## NUSAT 1 METHODS FOR RADAR FIELD STRENGTH MEASUREMENT

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**Abstract:** The methods for obtaining the gain pattern of an Air Traffic Control Radar using the data collected by NUSAT 1 are presented. The basic equations for this are given; these require an independently determined satellite attitude. In addition we show how simultaneous measurements made by separate channels can be related to obtain a single axis attitude, i.e., the direction of propagation of the incident radiation, and the power density of the radiation. This is done by solving a system of three algebraic equations.

**Introduction.** NUSAT 1 was the first satellite to be launched from a Getaway Special Cannister. It has six L-band channels which are driven by six orthogonally directed antennas. The purpose of these systems is to detect Air Traffic Control Radar (ATC) transmissions and measure the power density of the

incident radiation, thus allowing the gain pattern of the radar transmitter to be determined. Since the satellite is free floating (no attitude control) and since the gains of the satellite's receiving antennas are not unidirectional a fundamental problem arises in the determination of the direction of propagation (with respect to a satellite based coordinate system) of the incident radiation. To surmount this problem a crude attitude determination system was included in NUSAT 1. It will not give an attitude accurate enough for data reduction.

The purpose of this paper is twofold; one, we will present the equations needed to calculate the gain of the radar transmitter and power density of the radar wave. These can be used if an independently determined attitude is available. Two, we will also show how three simultaneous measurements made by three

separate channels can be related to obtain a single axis attitude; i.e., the direction of propagation of the incident radiation, and the power density of that radiation. This is done by solving a system of three algebraic equations. In addition, we will give a short error discussion to indicate the type of accuracy available.

1. Overview of NUSAT 1 For those readers who are unaware of the need and current methods for obtaining the power pattern of an ATC radar, we will give a brief summary of the situation; [1] contains more detailed information.

Air Traffic Control Radar The ATC radar transmitting antennas rotate at a rate of one revolution in four to ten seconds. They have an elevation beam width of about 40 degrees (the lower 15-20 degrees is used to detect aircraft) and an azimuth beam width of 2 degrees. They transmit on a frequency of 1030 Mhz. If the antenna is pointing too low, then ground interference causes nulls and lobes. So for safe operation in controlling air traffic, the radiation pattern for many different azimuthal orientations of the radar antenna is desired. In lieu of this, at least the location of the nulls and lobes.

The Satellite. The shape of NUSAT 1 is a twenty-six sided polyhedron. Upon launch, the satellite was not given any initial spin, nor is there any attitude control. Thus, the satellite's attitude is allowed to drift.

The satellite has a rather crude attitude determination system - eight symmetrically located wide angle field of view sensors (detecting radiation primarily in the visible region of the spectrum).

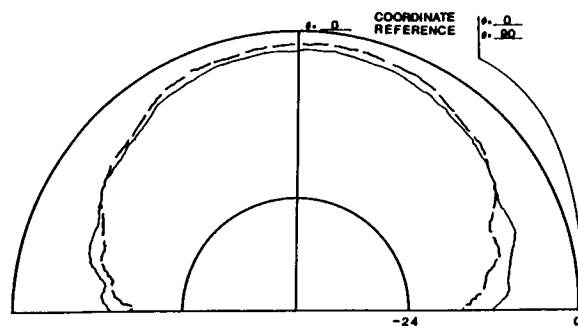


Fig. 1. Partial power gains in the horizontal  $G_h$  ---- and vertical  $G_v$  —.

The features of present interest to us are the six L-band antennas whose major axes are directed in six orthogonal directions. Each is a circularly polarized double Archimedes spiral antenna with a nominal total gain of 1.4 dB. One cut of the pattern (partial power gains in the horizontal and vertical directions) of a typical antenna is depicted in Fig. 1. Note that the usable range is a hemisphere centered about the antenna's major axis and that the gain is down by 6 or 7 dB at 60 degrees.

2. The Basic Equations. The calculation used to determine the gain of a radar antenna can be split into two steps. First, the measured value of the received power is used to find the power density of the incident wave at the satellite. Then this value is combined with data from orbital prediction routines to compute the gain of the radar. These calculations are based on the transmission equation.

The fundamental equation governing power transmission is the Friss Transmission Equation [2, Sec 2.17]:

$$P_r = (\lambda/4\pi r)^2 G_r(\theta_r, \phi_r) \times G_t(\theta_t, \phi_t) P_e P_t \quad (1)$$

where

$P_r$  = power supplied by the receiving antenna,

$P_t$  = power applied to the transmitting antenna,  
 $r$  = distance between the antennas,  
 $e$  = total efficiency,  
 $p$  = polarization loss factor,

$G_r(\theta_r, \phi_r)$  = isotropic gain of the receiving antenna in the  $(\theta_r, \phi_r)$  direction,  
 $G_t(\theta_t, \phi_t)$  = isotropic gain of the transmitting antenna in the  $(\theta_t, \phi_t)$  direction.

The relation between these parameters is shown in Fig. 2.

The power density  $W$  of the incident electromagnetic wave is related to the received power by

$$W = P_r 4\pi\lambda^{-2} (G_r(\theta_r, \phi_r) p)^{-1}. \quad (2)$$

By using this in equation (1), one can show that the desired quantity  $G_t(\theta_t, \phi_t)$ , the gain of the radar antenna-ground interference in the direction  $(\theta_t, \phi_t)$ , is

$$G_t(\theta_t, \phi_t) = 4\pi r^2 W / P_t e \quad (3)$$

In logarithmic decibel form this becomes

$$[G_t(\theta_t, \phi_t)]_{dB} = 20 \log r + [W]_{dB} + 10 \log(4\pi / e P_t),$$

where the last term may be dropped if the absolute gain is not desired and the quantities  $P_t$ ,  $e$  are nominally constant.

In our application,  $P_r$  is measured by the satellite; then equation (2) is used to calculate  $W$ . In (2) the angles  $\theta_r, \phi_r$  represent the direction of propagation of incident wave; i.e., the direction of the ATC radar in a satellite based coordinate system. These can be found from an attitude determination system or they can be computed simultaneously with  $W$ . In addition, the polarization loss factor,  $p$ ,

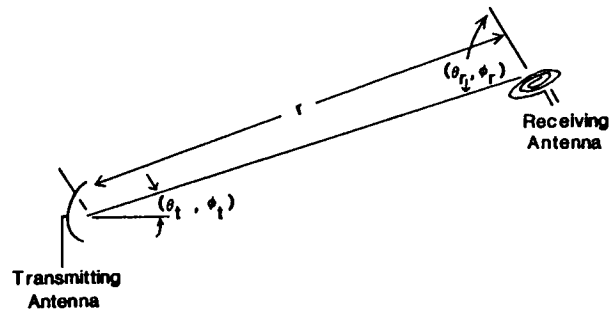


Fig. 2. Relation of parameters for power transmission.

must be evaluated. The evaluation of  $p$  and computation of  $W$  will be discussed in more detail in the next section. Once  $W$  is found, it can be used in (3) to get the gain of the radar antenna. Here, the variables  $r, \theta_t, \phi_t$  are the spherical coordinates of the satellite in a local coordinate system based at the radar antenna. These can be found from orbital prediction routines.

3. Computation of power density and the polarization loss factor. The power density  $W$  of the incident radar wave is computed using (2). Nominally, the radar wave is linearly polarized and the receiving antennas are circularly polarized; so  $p$  may be approximated by  $1/2$ . For more accuracy  $p$  needs to be evaluated; this may be done using the gain measurements performed on the receiving antennas.

The antennas were measured using a modified version of the Multiple-Amplitude-Component method. The entire gain pattern (letting  $\theta_r, \phi_r$  range over useful values, a hemisphere about the major axis) of each antenna was measured with a linearly polarized transmitter oriented at angles  $\psi$  of 0, 45, 90, and 135 degrees (see Fig. 3). This resulted in a measurement of the partial power gains  $G_\phi, G_{45}, G_\theta$ ,

$G_{135}$  respectively in the direction  $(\theta_r, \phi_r)$ .

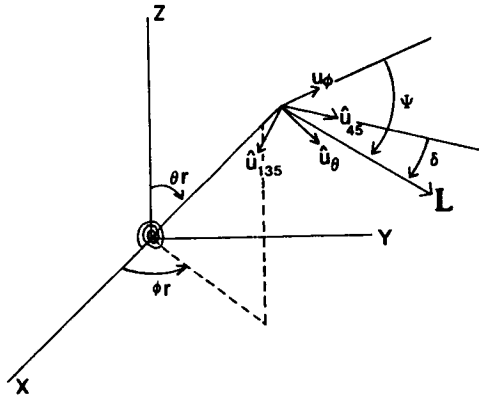


Fig. 3. Direction of polarization,  $L$ , of wave propagating from direction  $(\theta_r, \phi_r)$ .

Letting  $L$ , in Fig. 3, represent the electric field of the incident wave allows the polarization ratio to be related to either of the angles  $\psi$  or  $\delta$ . By using the angle  $\psi$  and, assuming that the wave is perfectly linear; i.e., that the power in the direction perpendicular to  $L$  is zero, one can show that

$$p = \cos^2\psi + \tau^2 \sin^2\psi + \tau \sin 2\psi \cos\beta / (1 + \tau^2)$$

where  $\tau, \beta$  are the polarization ratio and relative phase angle of the receiving antenna in the direction  $(\theta_r, \phi_r)$ . A similar equation is available for  $p$  using the angle  $\delta$ . Note that for a perfectly circular antenna we have  $\cos\beta=0$  and  $\tau=1$  so that  $p$  reduces to  $1/2$ . We also note that there is the option of approximating  $\cos\beta$  with a nominal value for each antenna. If this is done, only the measurements  $G_\phi, G_\theta$  need to be performed and

$$G_r p = G_\phi \cos^2\psi + G_\theta \sin^2\psi + \tau \sin 2\psi \cos\beta G_r / (1 + \tau^2).$$

Since a complete set of four measurements are available for NUSAT 1, both  $\tau$  and  $\beta$  can be found for each direction  $(\theta_r, \phi_r)$ . Omitting the intermediate steps, we get two different equations to use for the product  $G_r(\theta_r, \phi_r)p$  in equation (2). Using the angle  $\psi$  we get

$$G_r p = G_\phi \cos^2\psi + G_\theta \sin^2\psi + (1/2) \sin 2\psi (G_{45} - G_{135}). \quad (4)$$

Using the angle  $\delta$  we get

$$G_r p = G_{45} \cos^2\delta + G_{135} \sin^2\delta + (1/2) \sin 2\delta (G_\theta - G_\phi). \quad (5)$$

If the satellite's attitude can be found, then  $\theta_r, \phi_r$  can be determined from the relative location of the radar and the satellite at the time the measurement was made. Further,  $\psi$  and  $\delta$  can be found by applying the attitude matrix to a vector in the direction of the local vertical at the radar antenna. Thus,  $G_r(\theta_r, \phi_r)$  can be determined from either equation (4) or (5). For better accuracy, the equation that has the smallest value for  $\sin 2\psi$  or  $\sin 2\delta$  should be used (subtraction produces a loss of significance in the third terms).

Alternately, the power density,  $W$ , and the angles  $\theta_r, \phi_r$  can be determined simultaneously by relating three received power levels. For this though, the polarization loss factors must be approximated by  $1/2$ . Recall that NUSAT 1 has six L-band antennas oriented so that their major axes are directed in the six orthogonal directions. When the satellite is not in a null of the radar's radiation pattern at least three channels will be able to detect a signal. The geometry is shown in Fig. 4, where the major axes of the antennas which

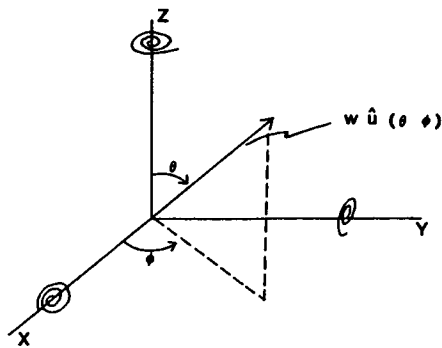


Fig. 4. Three L-band antennas and the direction of the incident wave with power density  $w$ .

are able to detect a particular signal have been placed along the  $x, y, z$  directions. From equation (2) we have for the three channels:

$$\begin{aligned} P_1 &= (\lambda^2/4\pi) G_1(\theta, \phi) p_1 W \\ P_2 &= (\lambda^2/4\pi) G_2(\theta, \phi) p_2 W \\ P_3 &= (\lambda^2/4\pi) G_3(\theta, \phi) p_3 W \end{aligned}$$

where  $G_i, p_i, p_i; i=1, 2, 3$  are associated with channel  $i$  and  $\theta, \phi$  give the direction of the radar in body coordinates. If the polarization loss factors are approximated by  $1/2$  this system can be solved for  $W, \theta, \phi$ . The gain functions  $G_i(\theta, \phi)$  are only known through test measurements, yet any of several numerical methods can be used to solve this system. If the chosen method requires the partial derivatives of the functions  $G_i$ , then it might be necessary to smooth these measurements. Actually this system should be solved in logarithmic decibel form:

$$\begin{aligned} [P_i]_{dB} &= 10 \log(\lambda^2/4\pi) \\ &+ [G_i(\theta, \phi)]_{dB} + [p_i]_{dB} \\ &+ [W]_{dB}, \quad i=1, 2, 3. \end{aligned}$$

Combining each  $[P_i]_{dB}$  with  $10 \log(\lambda^2/4\pi)$  and using  $P_i$  to denote the combined quantity this system becomes, with the dB subscripts dropped

$$\begin{aligned} P_1 &= G_1(\theta, \phi) + W + p_1, \\ P_2 &= G_2(\theta, \phi) + W + p_2, \\ P_3 &= G_3(\theta, \phi) + W + p_3. \end{aligned} \quad (6)$$

For actual computation, each  $p$  must be approximated by  $-3dB$ .

4. Preliminary error discussion. We now give an error discussion to indicate the origin of major errors and to realize the type of accuracy available. We will only give a short discussion of the errors as the complete error analysis is quite lengthy and is still in the process of being completed.

The gain of the radar antenna in the direction  $(\theta_t, \phi_t)$  is computed from (3) with the power density being calculated from (2). From (3), the relative error in  $G_t(\theta_t, \phi_t)$  is:

$$\begin{aligned} \Delta G_t/G_t &= 2\Delta r/r + \Delta W/W + \Delta e/e \\ &+ \Delta P_t/P_t. \end{aligned}$$

The angles  $\theta_t, \phi_t$  and the distance  $r$  are found from orbital prediction routines which are usually quite accurate. The efficiency  $e$  and power supplied to the transmitter  $P_t$  are close enough to being constant that they will also not contribute much error. The relative error in the power density  $\Delta W/W$  will be at least two orders of magnitude larger than the others. It can be found from (2),

$$\begin{aligned} \Delta W/W &= \Delta P_r/P_r + 2\Delta\lambda/\lambda + \\ &\Delta G_r p/G_r p. \end{aligned}$$

the received power  $P_r$  is obtained from measurements of the signal in the L-band channels. A calibration can be carried out for each channel, thus keeping  $\Delta P_r/P_r$  comparatively small. The relative error in the wave length  $\Delta\lambda/\lambda$  will be very small, the relative error in neglecting the doppler shift is .01 %. The term

$G_r(\theta_r, \phi_r)$  will produce the largest error.

An error analysis for equations (4) or (5) is in progress, but no results are available at this time. An indication of the error was obtained by considering  $G_r$  and  $p$  separately. If the angles  $\theta_r, \phi_r$  are found independently from  $W$ , then the error in  $G_r(\theta_r, \phi_r)$  will depend on the errors in  $\theta_r$  and  $\phi_r$ ; the accuracy of the gain measurements of the receiving antennas; and the variation of  $G$  with respect to  $\theta_r, \phi_r$ . If the angles  $\theta_r, \phi_r$  are known to be within .5 degrees, then  $\Delta G_r/G_r$  will be bounded by 12% since the gain measurements indicate that the variation is bounded by .25 dB/deg. Some quick estimates of the error in approximating  $p$  by 1/2 indicate that it will average about 1.2dB but may be as high as 2 dB.

Thus, the total error in finding the radar gain  $G_t(\theta_t, \phi_t)$  given values of  $\theta_r, \phi_r$  accurate to within .5 degrees and approximating  $p$  by .5 will be on the order of 1.6 dB but could be as much as 2.3 dB.

To give a preliminary indication of the error in determining  $\theta_r, \phi_r$  and  $W$  simultaneously, the author worked several examples. These gave the following bounds:

$$\begin{aligned}\Delta\theta, \Delta\phi &\leq 10 \text{ degrees,} \\ \Delta W &\leq 1 \text{ dB.}\end{aligned}$$

5. Conclusion As yet the satellite has not made any actual radar measurements. Therefore, no test results are available for presentation. But, as reproduced here, the necessary equations are in place for calculating the ATC radar gain in case some measurements are made. These same equations will be needed for the second satellite to attempt an ATC radar calibration, NUSAT2.

## References

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